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HORIZONTAL FORCES DUE TO WAVES ACTING ON LARGE VERTICAL CYLINDERS IN DEEP WATER

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San Diego, California

October 1972

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SUMMARY

PROBLEM

To determine the forces due to waves on the vertical buoyancy columns of floating stable ocean platforms.

RESULT

By means of the method described, forces may be calculated with greater confidence.

RECOMMENDATION

Horizontal forces due to waves acting on large vertical cylinders in deep water should be calculated by means of the methods of this report.

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13 ABSTRACT
The special case of horizontal wave forces on large vertical cylinders in deep water is considered. The typical application for such a case is the calculation of horizontal forces on column-stabilized floating ocean platforms. Existing literature discussing horizontal wave forces on cylinders does not generally agree on how to predict these forces. Since for large-diameter cylinders in deep water the maximum force is completely inertial, the problem of deriving a solution is considerably simplified. In this study, an expression for the maximum horizontal wave force on large-diameter circular cylinders mounted vertically in deep water has been analytically derived. Experimental model studies were also conducted and the resulting measured forces were within 20 percent of predicted forces. An example of how to predict horizontal wave forces using the methods of this report is given.

I

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INTRODUCTION

Wave forces on vertical cylinders are due to both viscous and inertial effects. The problem appears to be considerably simplified when the forces are predominantly inertial. In general, the inertial forces predominate as cylinder diameter and water depth increase. A column-stabilized floating ocean platform presents such a case. The general arrangement to be considered is a cylinder extending into the water some distance *B* below the surface (Fig. 1). Several of these cylinders would be connected together structurally a few diameters apart to form a platform about 30 ft above the still water level. Although only single cylinders are considered in this report, the method is applicable to groups of cylinders also. The method of finding the inertial coefficient for a group of closely spaced cylinders is given in the cited report by Dalton and Helfinstine.

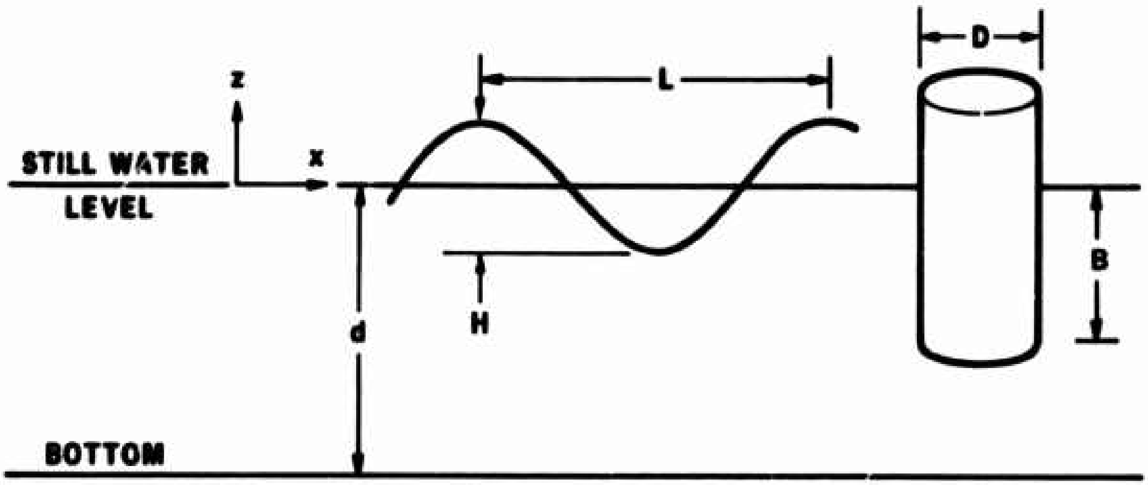

Figure 1. Coordinate system and some nomenclature.

BACKGROUND

In the last twenty years, many papers have been published dealing with wave forces on cylinders. The Bibliography presented in this report alone lists 31 papers on the subject. A discussion of some of the papers follows.

Basic to any study of wave forces on stationary objects are J. R. Morison's two papers.* The first paper is a preliminary report of the second. In these papers, it is

*See Bibliography for all cited references.

proposed that the force consists of two components: a drag force and an inertial force. The drag force is proportional to the fluid density, the projected area, and the square of the fluid particle velocity. The inertial force is proportional to the fluid density, the volume of the object, and the fluid particle acceleration. These two components are added together to give the "Morison equation":

$$f = \frac{1}{2} C_d \rho A u^2 + C_m \rho V \dot{u}$$

The coefficients C_d and C_m are determined by experiment, taking advantage of the fact that the two components are out of phase. As the wave passes the cylinder, the particle velocity u is zero at the still water level and C_m can be determined. The particle acceleration $\dot{u}$ is zero at the crest and trough and C_d can be determined. Morison found that the relative importance of the inertial force increases for deep water (d/L large) or for large cylinders in small waves (D/H large).

Laboratory tests to determine C_d and C_m were conducted on circular cylinders, H-sections, and flat plates. No trend was found in C_d or C_m as a function of d/L, H/L, or Reynolds number. In addition, ocean tests were conducted on a 3-1/2-in.-diameter cylinder at Monterey, California. Because of the small value of d/L (less than 0.06), the inertial force was not a factor. Values of C_d ranged from 0.24 to 2.72 in the ocean tests. One of Morison's conclusions is that the theoretical value of 2.0 for C_m seems adequate, but more work needs to be done to correlate C_d over the range of variables.

Another approach to estimating wave forces on circular cylinders was suggested by Iverson and used by Crooke. Iverson suggested that the resistance of objects moving in accelerated motion could be given by an equation of the same form as that used for steady motion:

$$f = C \frac{1}{2} \rho A u^2$$

However, in accelerated motion, C is a function of Reynolds number, uD/ν; Froude number, u^2/gD; geometry; and Iverson's modulus, $\dot{u}D/u^2$. Iverson experimentally obtained good correlation of C versus $\dot{u}D/u^2$ for completely submerged flat disks accelerated perpendicular to the plane of the disk. Disks were chosen because the steady-state drag coefficient has been found to have a constant value (1.12) above Reynolds numbers of 10^3. In addition, the Froude number does not influence the resistance coefficient of completely submerged objects.

Crooke applied Iverson's method to Morison's laboratory data on circular cylinders. Even though the cylinders were only partially submerged, and resistance coefficient was known to be strongly dependent on Reynolds number for circular cylinders, Crooke still obtained a good correlation for C versus $\dot{u}D/u^2$. Perhaps this was because the steady-flow resistance coefficient is constant over the small range of Reynolds numbers (2×10^3 to 10^4) involved in Morison's data. R. L. Wiegel (1964) combined his own experimental data,

Crooke's data and Keim's data, and obtained a poor correlation of C versus $\dot{u}D/u^2$. The data cover a wider range of Reynolds numbers (2×10^3 to 10^6).

Wiegel's paper (1957) is also significant in the study of wave forces on cylinders, and much of it has been incorporated into one chapter of his book. Tests were conducted in about 50 ft of water near Davenport, California, using 6.625-, 12.75-, 24-, and 60-in.-diameter piles. The values of C_d and C_m were computed. No relationship was found for C_d versus Reynolds number for the Reynolds number range of 3×10^4 to 9×10^5. The average value of the coefficient of mass was 2.5 with a normal Gaussian distribution. When average values of C_d and C_m were used, the maximum forces were predicted within ±100 percent.

Another method for predicting wave forces on circular cylinders was developed from electromagnetic diffraction theory applied to water waves by R. C. MacCamy and R. S. Fuchs. For the case of "small" piles ($D < 0.1L$), the force predicted by the diffraction theory was the same as Morison's inertial force when C_m is 2.0. Diffraction theory showed very good agreement when applied to Morison's laboratory data for low waves in deep water. The data available from these reports have been summarized in Table 1.

Table 1. Summary of Coefficients

Source	D (ft)	d (ft)	Maximum Reynolds Number From	Maximum Reynolds Number To	Average C_d	Average C_m
Morison, Laboratory (1954)	0.083	2	2×10^3	3×10^5	1.6	1.5
Wiegel, Ocean (1957)	0.5, 1, 2	50	3×10^4	9×10^5	0.6	2.5
Jen, Laboratory (1968)	0.5	3	5×10^3	2×10^4	...	2.04
			Irregular waves No. 1		...	2.20
			Irregular waves No. 2		...	2.08
Evans, et al., Ocean (1969)	2, 3, 4	33	10^4	6×10^7	0.585	1.5
	3	100	10^4	6×10^7	0.88	1.76
Steady flow	.....	...	2×10^2	3×10^5	1.2	0.0
Hydrodynamic theory	.....	...	.....	.....	0.0	2.0
Diffraction theory	.....	...	.....	.....	0.0	2.0

Some other conclusions in the published literature have also been found useful for this study. The relative importance of inertial forces was found to increase as water depth and pile diameter increased. Using Morison's data, MacCamy and Fuchs found very good agreement between calculated and experimental forces when the forces were predominantly inertial. Keulegan and Carpenter, in their report on forces on cylinders and plates in an oscillating fluid, found that when their "period parameter" uT/D is small, the agreement

between observed and computed forces was satisfactory. When uT/D is small the forces are predominantly inertial. Lappo, using extensive laboratory and ocean data obtained in the USSR, found calculated and experimental forces differed by less than 10 percent when the forces were primarily inertial.

ANALYSIS

From the discussion of existing literature, it appeared worthwhile to examine when inertial forces predominate and what the magnitude of these inertial forces might be. The Morison equation could be utilized, but expressions for the water particle velocity and acceleration are first needed. Airy theory gives:

$$u = \frac{\pi H}{T} \frac{\cosh[(2\pi/L)(d + z)]}{\sinh 2\pi d/L} \cos\theta$$

and

$$\dot{u} = \frac{-2\pi^2 H}{T^2} \frac{\cosh[(2\pi/L)(d + z)]}{\sinh 2\pi d/L} \sin\theta$$

where $\theta = 2\pi t/T$ and $t = 0$ at the crest. These simple equations seem to give a good description of the water particle velocity and acceleration, provided the relative height of the wave is not too great. Furthermore, as d becomes large:

$$\frac{\cosh[(2\pi/L)(d + z)]}{\sinh 2\pi d/L} \longrightarrow \exp(2\pi z/L)$$

So, for deep water:

$$u = \frac{\pi H}{T} \exp(2\pi z/L) \cos\theta$$

and

$$\dot{u} = \frac{-2\pi^2 H}{T^2} \exp(2\pi z/L) \sin\theta$$

These can be substituted into the Morison equation to give:

$$\frac{dF}{dz} = \frac{1}{2} C_d \rho D \frac{\pi^2 H^2}{T^2} \exp(4\pi z/L) \cos^2\theta - \frac{1}{2} C_m \rho \pi^3 \frac{D^2 H}{T^2} \exp(2\pi z/L) \sin\theta \qquad (1)$$

This equation can be integrated to get the total force on a vertical cylinder. The limits of integration are from $z = 0$ at the still water level to $z = -\infty$ at the bottom. With the substitution of the deep-water identity $L = (g/2\pi)T^2$, the result is:

$$F = \frac{1}{16} C_d \gamma D H^2 \cos^2\theta - \frac{1}{8} C_m \pi \gamma D^2 H \sin\theta$$

The maximum force is found by differentiating with respect to θ and setting the result equal to zero:

$$\frac{dF}{d\theta} = (C_d H \sin\theta - C_m \pi D)\cos\theta = 0$$

There are two possible solutions:

$$\sin\theta = \pi \frac{C_m D}{C_d H}$$

or, if $\pi(C_m D/C_d H) > 1$, $\cos\theta = 0$. Now, when $\cos\theta = 0$, the force is completely inertial. So the condition that the force be predominantly inertial is that $\pi(C_m D/C_d H)$ be greater than one. Viscous forces still exist; but at the time the horizontal force is a maximum, it is completely inertial, and the maximum force will then be:

$$F = \frac{\pi}{8} C_m \gamma D^2 H \tag{2}$$

This is the result for a cylinder extending from the surface to the bottom. For a cylinder extending a distance B down from the surface, Eq. (1) can be integrated from $z = 0$ to $z = -B$ to give:

$$F = [1 - \exp(-2\pi B/L)] \frac{\pi}{8} C_m \gamma D^2 H \tag{3}$$

Then let $\zeta = [1 - \exp(-2\pi B/L)]$, and $C_m = 2$, the value obtained from hydrodynamic theory, and the maximum force is:

$$F = 50\zeta D^2 H \tag{4}$$

The factor ζ has been calculated and is shown graphically in Fig. 2.

The distance Z_f, from the still water level to where the resultant force acts, can be determined using Eqs. (1) and (3):

$$Z_f = \int_0^{-B} \frac{z\,dF}{F}$$

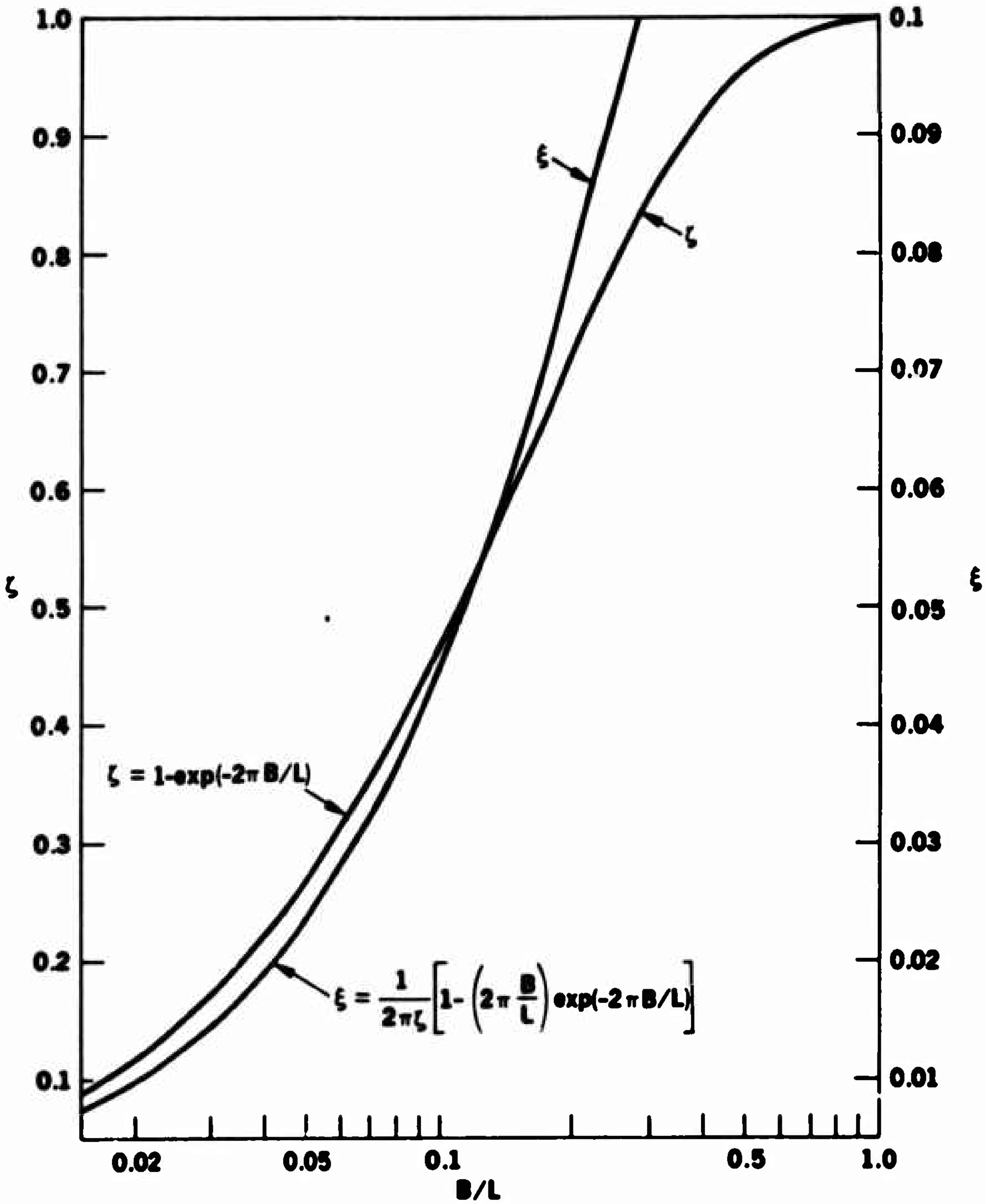

Figure 2. The force and distance coefficients ζ and ξ.

The result is:

$$Z_f = \frac{L}{2\pi\zeta}\left[1 - \left(2\pi\frac{B}{L} + 1\right)\exp(-2\pi B/L)\right] \qquad (5)$$

Then let

$$\xi = \frac{1}{2\pi\zeta}\left[1 - \left(2\pi\frac{B}{L} + 1\right)\exp(-2\pi B/L)\right]$$

and

$$Z_f = \xi L \qquad (6)$$

The factor ξ has been calculated and is also shown graphically in Fig. 2.

EXPERIMENTAL MODEL STUDIES

Model studies were conducted in the model basin at Offshore Technology Corp., Escondido, California. Water depth for the tests was 13.2 ft. The cylinders were attached to a parallel bar arrangement over the basin and extended 3 ft into the water (Fig. 3). The cylinders were open at the bottom. Forces were measured by strain gages mounted on a stainless steel ring connecting the movable cylinder to the rigid structure. The force measurement was calibrated with weights before the tests. Wave height was measured with a capacitance-type wave staff. Wave period was determined by the oscillator setting of the wave maker. The wave shapes were checked photographically and follow the relationship $L = 5.12T^2$. The basin contained fresh water at ambient temperature. The range of test parameters in the experimental study is shown in Table 2. Over 200 data points were obtained.

DISCUSSION

To obtain a good visual comparison, the forces found experimentally were divided by ζ and plotted against D^2H (Fig. 4). The graphical representation of Eq. (4) is also shown. All of the experimental data fall within 20 percent of the value predicted by Eq. (4). Some of this scatter could have been caused by errors in the measuring system or binding in the linkages. Even so, the experimental data show good agreement with the analytical prediction when compared with the general case where the maximum force is both viscous and inertial. One reason for this good agreement is the elimination of the drag coefficient as a direct variable in predicting the maximum force. The drag coefficient for unsteady motion is a difficult coefficient to determine for the general case.

On the basis of the analytical expressions derived for the magnitude and distribution of the maximum forces, supported by the experimental data, some general guidelines can be

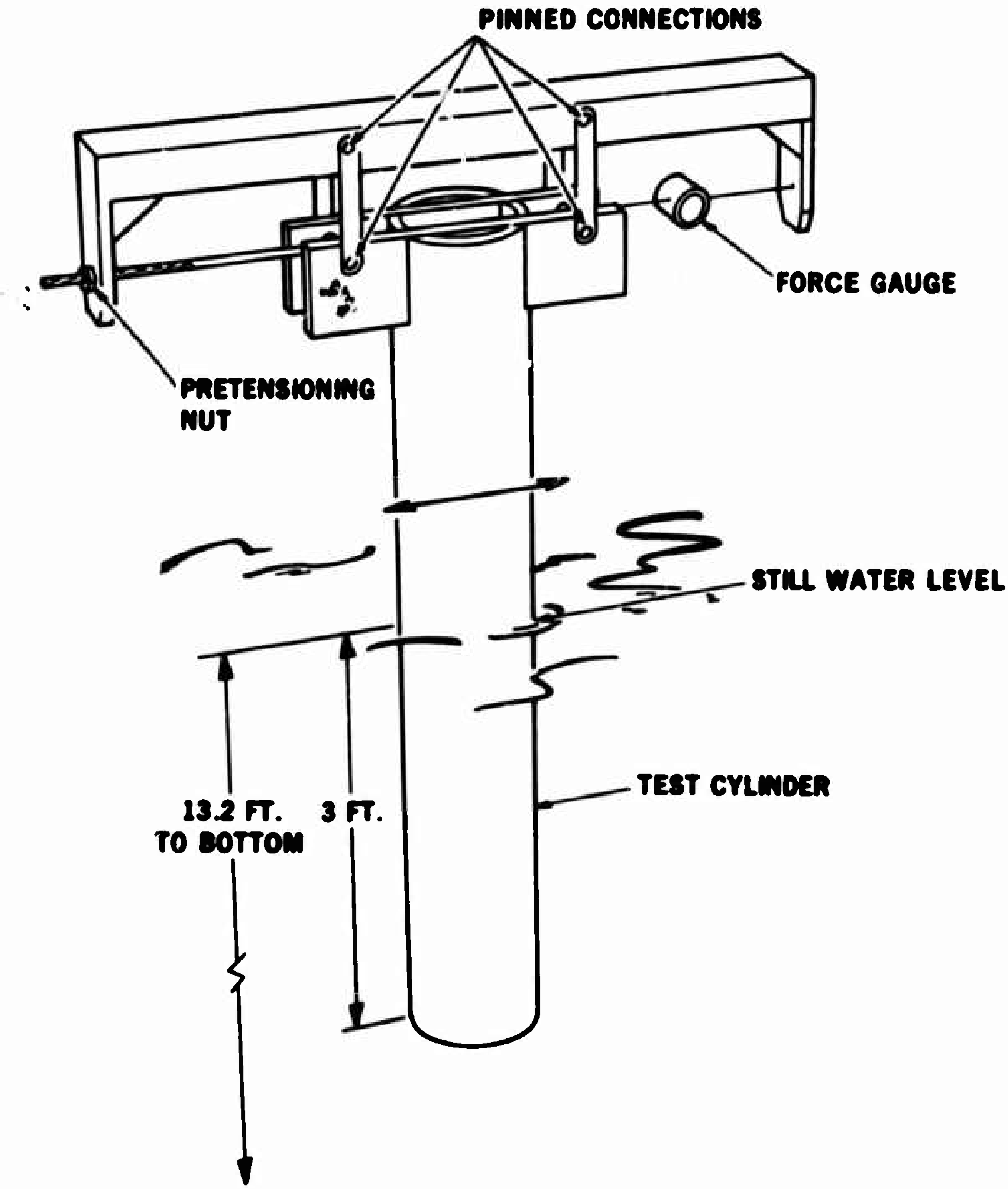

Figure 3. Test arrangement for measuring horizontal forces in the wave basin.

Table 2. Model Test Parameters.

Parameter*	Range	
	From	To
D/H	0.22	3.35
D/L	0.0078	0.13
D/d	0.02	0.05
d/H	11.7	65.6
d/L	0.41	2.5
H/L	0.02	0.08
R_e max	3.6×10^4	1.6×10^5

*d = 13.125; D = 0.25, 0.5, 0.67; B = 3 ft.

formed for the benefit of engineers interested in the horizontal wave forces on the vertical buoyancy cylinders of stable ocean platforms.

1. The maximum horizontal force due to waves is proportional to the square of the cylinder diameter when the maximum force is inertial.

2. The force distribution is concentrated near the surface. In fact, Fig. 2 shows that about one-half the possible maximum force on the cylinder occurs in the first one-tenth wavelength of depth from the water surface. For the first one-tenth wavelength (200 ft for a 20-sec wave) of depth, the force distribution is almost linear with depth.

3. In the range of interest for platform buoyancy columns, the horizontal forces due to waves are about proportional to the column length and the square of the diameter. Therefore, wave force considerations should not be a factor in column proportioning since, for a given column buoyancy, the force would be about the same regardless of the length-to-diameter ratio.

4. However, the magnitude and distribution of the wave forces on the vertical cylinders do need to be known for the structural design.

AN EXAMPLE

A 20-ft-diameter cylinder is fixed vertically in 1000 ft of water. The cylinder extends 100 ft into the water. The design wave for the area has a double amplitude of 40 ft and an 18-sec period. So, for this example, B = 100, d = 1000, D = 20, H = 40, and T = 18.

First, is the cylinder in "deep water"? In other words, is d/L greater than 0.5? The wavelength is calculated from $L = 5.12T^2$ and is 1658 ft. Then d/L = 1000/1658 = 0.605 and the cylinder is in "deep water."

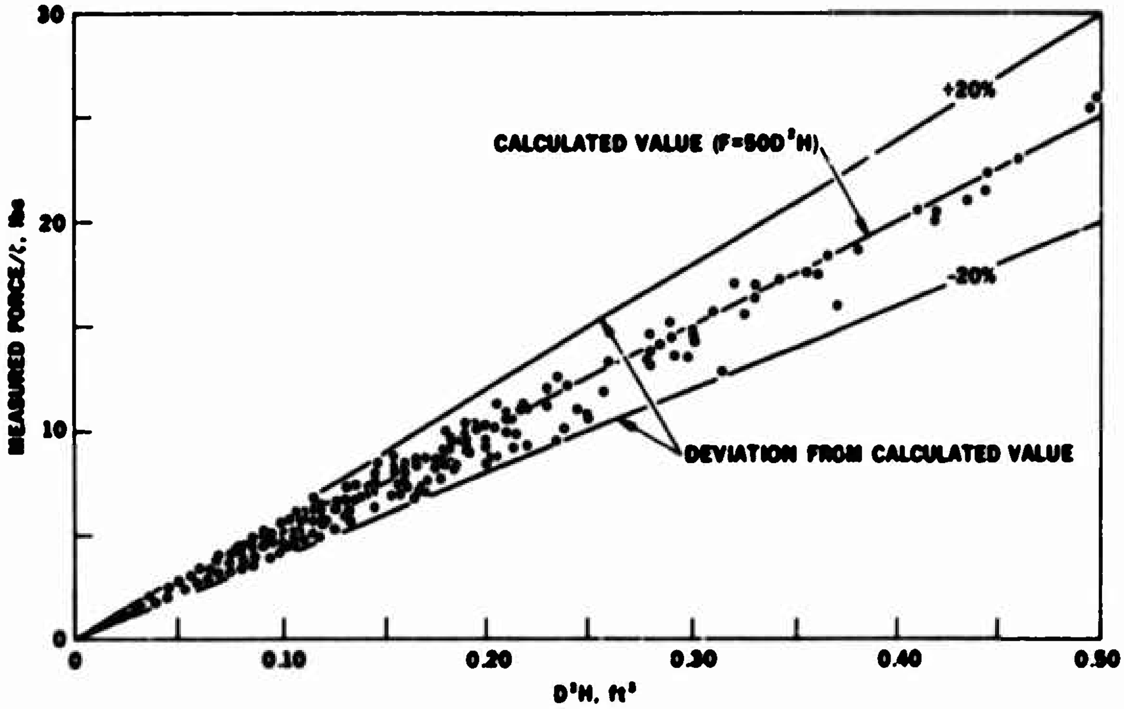

Figure 4. Comparison of experimentally measured and analytically calculated forces.

Second, is the maximum force completely inertial? In other words, is D/H greater than 0.16? $D/H = 20/40 = 0.5$ and the maximum force is completely inertial.

Third, since the cylinder extends only 100 ft into the water, what is the fraction of force? $B/L = 100/1658 = 0.0604$ and the force fraction, ζ, is 0.32 from Fig. 2.

Then the maximum force on the cylinder is given by Eq. (4):

$$F = 50\zeta D^2 H$$

$$F = (50)(0.32)(400)(40)$$

$$F = 256{,}000 \text{ lb}$$

Also, from Fig. 2 for $B/L = 0.0604$, $\xi = 0.0285$ and the point where the resultant force acts is given by Eq. (6):

$$Z_f = \xi L$$

$$Z_f = (0.0285)(1658)$$

$$Z_f = 47.25 \text{ ft below the still water level}$$

SUMMARY

1. At this time there does not appear to be an exact general method for predicting horizontal wave forces on vertical cylinders to better than ±100 percent accuracy. However, if the forces are predominantly inertial, good agreement has been found between predicted and measured forces.

2. It was found analytically that, for deep water ($d > L/2$), the maximum force would be completely inertial if $\pi(C_m D/C_d H) > 1$. Assuming C_m/C_d is 2, then D/H must be greater than 0.16.

3. The maximum horizontal force is then found analytically to be given by:

$$F = [1 - \exp(-2\pi B/L)] \frac{\pi}{8} \gamma C_m D^2 H \qquad (3)$$

With the resultant acting at:

$$Z_f = \frac{L}{2\pi\zeta}\left[1 - \left(2\pi \frac{B}{L} + 1\right) \exp(-2\pi B/L)\right] \qquad (5)$$

Figure 2 may be used as an aid for solving the above equations.

4. Model studies were conducted and the maximum horizontal force was predicted to within 20 percent by Eq. (4).

NOMENCLATURE

A	Projected area per unit length, ft
B	Length of cylinder below still water level, ft
C	Coefficient for use with Iverson's method
C_d	Drag coefficient
C_m	Inertial coefficient
d	Water depth, ft
D	Diameter, ft
f	Force per unit length, lb/ft
F	force, lb
g	Gravitational constant, 32.2 ft/sec^2
H	Wave height, ft
L	Wavelength, ft
t	Time, sec (t = 0 at the wave crest)
T	Wave period, sec
u	Horizontal velocity, ft/sec
$\dot{u}$	Horizontal acceleration, ft/sec^2
V	Volume per unit length, ft^2
x	Horizontal distance, ft
z	Distance from still water level, ft (negative downward)
Z_f	Distance from still water level to where resultant force acts, ft

γ Specific weight, lb/ft^3 ($\gamma = \rho g$); for seawater, $\gamma = 64$

ζ $1 - \exp(-2\pi B/L)$ (Fig. 2)

ν Kinematic viscosity, ft^2/sec

ξ $\frac{1}{2\pi\zeta}\left[1 - \left(2\pi\frac{B}{L} + 1\right)\exp(-2\pi B/L)\right]$ (Fig. 2)

π 3.14

ρ Density, slugs/ft^3 ($\rho = \gamma/g$); for seawater, $\rho = 2$

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